

Roll No. 2811162

Total Pages : 4

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### MATHEMATICS-III

Paper-MATH-201E

Time Allowed : 3 Hours]

[Maximum Marks : 100

**Note** : Attempt five questions in all, selecting at least one question from each Unit. All questions carry equal marks.

#### UNIT-I

1. (a) Obtain Fourier series expansion of

$$f(x) = \left( \frac{\pi - x}{2} \right)^2 \text{ for the range } (0 \text{ to } 2\pi).$$

- (b) Find Fourier series to represent  $f(x)$  given by

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \leq x \leq 0 \\ 1 - \frac{2x}{\pi}, & 0 \leq x \leq \pi \end{cases}$$

Deduce that

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}.$$

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2. (a) Find Fourier transform of

$$f(x) = \begin{cases} x^2 & |x| < a \\ 0 & \text{otherwise} \end{cases}$$

- (b) Use Fourier sine transform to solve the equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad (x > 0, t > 0)$$

subject to the conditions :

- (i)  $u(0, t) = 0$ ,  
 (ii)  $u(x, t)$  is bounded,  
 (iii)  $u(x, 0) = \begin{cases} 1 & 0 < x < 1 \\ 0 & x \geq 1 \end{cases}$

#### UNIT-II

3. (a) If  $f(x)$  is an analytic function with constant modulus, show that  $f(z)$  is constant.

- (b) Find the regular function, whose imaginary part is

$$v = e^{-x}(x \sin y - y \cos y).$$

4. (a) Find the bilinear transformation which maps the points  $z = -1, i, 1$  of the  $z$ -plane onto  $w = 1, i, -1$  of the  $w$ -plane respectively.

- (b) Under the transformation  $w = \frac{1}{z}$ , find the image of

$$|Z - 2i| = 2$$

#### UNIT-III

5. (a) Let A and B be two events with their probabilities

$$P(A) = \frac{1}{2}, \quad P(B) = \frac{1}{3} \quad \text{and} \quad P(A \cap B) = \frac{1}{4}, \quad \text{Find}$$

$$P(A \cup B), \quad P(A/B) \text{ and } P(A/B^c).$$

- (b) In a certain college, 4% of the boys and 1% of girls are taller than 1.8 m. Further more 60% of the students are girls. If a student is selected at random and is found to be taller than 1.8 m, what is the probability that the student is a girl?

6. (a) If 10 percent of the rivets produced by a machine are defective, find the probability that out of 5 rivets chosen at random (i) none will be defective, (ii) one will be defective and (iii) atleast two will be defective.

- (b) For a normally distributed variate with mean 1 and S.D. 3, find the probabilities that :

$$(i) \quad 3.43 \leq x \leq 6.19, \quad (ii) \quad -1.42 \leq x \leq 6.18.$$

#### UNIT-IV

7. (a) Using graphical method, solve

$$\text{Min } Z = 20x + 30y$$

subject to  $x + 2y \leq 40, 2x + y \leq 30,$

$$4x + 3y \geq 60, x, y \geq 0.$$

(b) Use dual simplex method to solve

$$\text{Maximize } Z = -3x_1 - x_2$$

subject to

$$x_1 + x_2 \geq 1, \quad 2x_1 + 3x_2 \geq 2; \quad x_1, x_2 \geq 0.$$

8. What are the feasible solution and optimal solution of a L.P.P. ?  
Using simplex method, find the optimal feasible solution of the following L.P.P. :

$$\text{Minimize } Z = x_1 - 3x_2 + 2x_3$$

$$\text{Subject to } 3x_1 - x_2 + 2x_3 \leq 7, \quad -2x_1 + 4x_2 \leq 12,$$

$$-4x_1 + 3x_2 + 8x_3 \leq 10, \quad x_1, x_2, x_3 \geq 0.$$